Domain 5 • Lesson 28

Probability

Common Core Standards:
7.SP.5, 7.SP.6, 7.SP.7.a, 7.SP.7.b

Getting the Idea

Probability measures the chance of an event happening based on the number of the **possible outcomes**. Probability can be expressed as a fraction or a decimal from 0 to 1. A probability close to 0 means an event is unlikely. A probability close to 1 means an event is very likely. A probability close to $\frac{1}{2}$ or 0.5 means an event is neither unlikely nor likely. You can also express a probability as a percent.

The **theoretical probability** of an event is the ratio of the number of ways the event can occur **(favorable outcome)** to the number of possible outcomes. The probability, P, of an event, A, is:

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.$$

Example 1

Josh is going to choose a random card from 13 cards. The cards are numbered from 1 to 13. What is the probability that he will choose a card with a number less than 5? Determine if the event is likely, unlikely, or neither.

Strategy **Find the theoretical probability.**

Step 1 Count the number of favorable outcomes.

There are 4 cards (1, 2, 3, 4) with a number less than 5.

Step 2 Count the number of possible outcomes.

There are a total of 13 cards, each with the same chance of being drawn.

Step 3 Find the theoretical probability.

$$P(\text{card with a number less than 5}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{4}{13}$$

Step 4 Determine if the event is likely or unlikely.

$\frac{4}{13}$ is closer to 0 than it is to 1, and it is less than $\frac{1}{2}$. So, the event is unlikely.

Solution **The probability of choosing a card with a number less than 5 is $\frac{4}{13}$. The event is unlikely.**

You can use theoretical probability to make a prediction. Multiply the theoretical probability by the number of **trials**, or times the experiment is performed, to predict the number of favorable outcomes.

Example 2

Peter will roll a number cube, labeled 1 through 6, a total of 90 times. What is a good prediction for the number of times that the number cube will land on 5?

Strategy **Find the number of possible outcomes and favorable outcomes.**

Step 1 Find the number of possible outcomes.

There are 6 possible outcomes for the number cube.

Step 2 Find the number of favorable outcomes.

There is one 5 on the number cube.

Step 3 Write the theoretical probability in simplest form.

$$P(\text{rolling a 5}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$$

Step 4 Multiply the probability by the number of trials.

$$\frac{1}{6} \times 90 = 90 \div 6 = 15$$

Solution **A good prediction is that Peter will roll a 5 about fifteen times.**

Experimental probability is the ratio of the total number of times the favorable outcome happens to the total number of trials, or times the experiment is performed. The experimental probability, P_e, of event A is:

$$P_e(A) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}}.$$

Experimental probability is useful when you need to make predictions about an event. As the number of trials increases, the experimental probability gets closer to the theoretical probability.

Example 3

Minnie conducted an experiment with a spinner. The results are shown in the table.

Number	1	2	3	4	5	6
Times Landed	10	7	6	8	5	6

Based on the data, what is the probability that the spinner will land on 2 on the next spin?

Strategy **Find the experimental probability.**

Step 1 Find the number of trials.

$10 + 7 + 6 + 8 + 5 + 6 = 42$

Step 2 Find the number of favorable outcomes.

The spinner landed on 2 a total of 7 times.

Step 3 Write the experimental probability as a fraction in simplest form.

$\frac{7}{42} = \frac{1}{6}$

Solution **The experimental probability of the spinner landing on 2 on the next spin is $\frac{1}{6}$.**

Example 4

Gavin rolls a number cube, labeled 1 to 6, a total of 40 times. The number 4 is rolled 8 times. What is the experimental probability of rolling a 4? What is the theoretical probability? Describe the difference between the two.

Strategy **Use the formulas for experimental probability and theoretical probability.**

Step 1 Find the experimental probability.

The total number of trials is 40.

The number 4 is rolled 8 times, so the number of favorable outcomes is 8.

$P_e(4) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}} = \frac{8}{40} = \frac{1}{5}$

Step 2 Find the theoretical probability.

The number of possible outcomes is 6.

There is only one 4 on a number cube, so the number of favorable outcomes is 1.

$P_e(4) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$

Step 3 Compare the experimental probability and the theoretical probability.

The experimental probability is $\frac{1}{5}$, and the theoretical probability is $\frac{1}{6}$.

The theoretical probability shows the outcome you would expect.

The experimental probability shows the outcome that actually occurred during the experiment.

Solution **The experimental probability of rolling a 4 is $\frac{1}{5}$. This is greater than the theoretical probability of $\frac{1}{6}$.**

Coached Example

The Skate Pro Company manufactures skateboards. They found 12 defective skateboards in a batch of 400. How many defective skateboards might they find in a batch of 1,200?

Find the experimental probability of a defective skateboard in simplest form.

There were ________ defective skateboards in a batch of ________.

Write the probability and express it in simplest form.

P(defective skateboard) = ________ = _______

Express the probability as a decimal. ________

Multiply the probability by 1,200.

1,200 × ________ = ________

The Skate Pro Company might expect to find ________ defective skateboards in a batch of 1,200.

Lesson Practice

Choose the correct answer.

Use the following event for questions 1 and 2.

Dan rolled a number cube 20 times. The cube landed on the number 3 six times.

1. What is the experimental probability that Dan will roll a number 3 the next time he rolls the number cube?

A. $\frac{3}{10}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{7}{10}$

2. Which best describes what would likely happen if Dan rolled the number cube another 80 times?

A. There would be no change.

B. The experimental probability would get farther from the theoretical probability.

C. The experimental probability would exactly match the theoretical probability.

D. The experimental probability would get closer to the theoretical probability.

3. Sonya wrote each letter of LEDBETTER on a separate index card and put the cards in a box. She picked one letter at random, put the card back, and then repeated the experiment. If she performed this experiment 90 times, which is the best prediction for the number of times that Sonya would pick a T?

A. 10

B. 20

C. 25

D. 30

4. A lightbulb manufacturer found that out of 200 lightbulbs, 15 were defective. How many lightbulbs should the manufacturer expect to be defective out of 2,400 lightbulbs?

A. 40

B. 80

C. 120

D. 180

5. Shia conducted an experiment with a spinner. The results are shown in the table below.

Number	1	2	3	4
Times Landed	8	4	5	3

Based on this data, how many times can Shia expect to spin a 1 in the next 20 spins?

A. 8

B. 7

C. 5

D. 3

6. There are 12 girls and 8 boys in Ms. Sander's class. Each day, she randomly asks one student to take attendance. In 180 school days, which is the best prediction for the number of times that the student will be a girl?

A. 72

B. 90

C. 99

D. 108

7. Blake tossed a coin 80 times. The coin landed on heads 60 times.

A. What is the experimental probability that Blake will toss heads? Show your work.

____

____

B. Is the experimental probability greater than or less than the theoretical probability? Explain your thinking.

____

____

____

8. A random card is chosen from cards that are numbered from 1 to 20. Which probability is unlikely? Circle all that apply.

A. the card chosen is 1

B. the card chosen is a composite number

C. the card chosen is an even number

D. the card chosen is a multiple of 7

E. the card chosen is divisible by 6

9. Evelyn rolls a number cube, labeled 1 through 6. Write each probability of an event in the correct box.

P(rolling a 1)	P(rolling a prime number)	P(rolling a number $>$ 1)	P(rolling a number $>$ 5)	P(rolling an even number)

Unlikely	Neither Unlikely Nor Likely	Likely

10. William wrote each letter of INSTITUTION on a separate index card and put the cards in a box. He picked one letter at random, put the card back, and then repeated the experiment. Circle the number that makes the statement true.

If William performed this experiment 110 times, the best prediction for the number of times that he would pick an I is [11 / 30 / 33].

11
30
33

11. Sam conducted an experiment with a spinner. The results are shown in the table. Draw a line from each experimental probability to its value.

Color	Red	White	Blue	Green	Yellow	Orange
Times Landed	24	30	23	25	21	27

A. P(land on Red) • • $\frac{1}{5}$

B. P(land on Yellow) • • $\frac{9}{50}$

C. P(land on Orange) • • $\frac{7}{50}$

D. P(land on White) • • $\frac{4}{25}$

12. A ping-pong ball manufacturer found that out of 400 ping-pong balls, 25 were defective. Should the manufacturer expect each number of defective ping-pong balls given the number made? Select Yes or No.

A. 5 defective out of 80 made ○ Yes ○ No
B. 15 defective out of 250 made ○ Yes ○ No
C. 35 defective out of 600 made ○ Yes ○ No
D. 45 defective out of 720 made ○ Yes ○ No
E. 60 defective out of 960 made ○ Yes ○ No

13. Courtney rolled a number cube, labeled 1 to 6, a total of 80 times. The number 2 was rolled 18 times. Use an item from the box to make each statement true.

The experimental probability of rolling 2 was ____________.

The theoretical probability of rolling 2 was ____________.

The experimental probability was ____________ than the theoretical probability.

$\frac{1}{7}$	$\frac{7}{120}$
$\frac{1}{6}$	less
$\frac{9}{40}$	greater